Carleton UNIVERSITY
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Mathematics Tutorial Series
Calculus

What you need to know about limits

First we tend to abbreviate “limit” as “lim”.

Most limits can be reduced to one of the following two types.

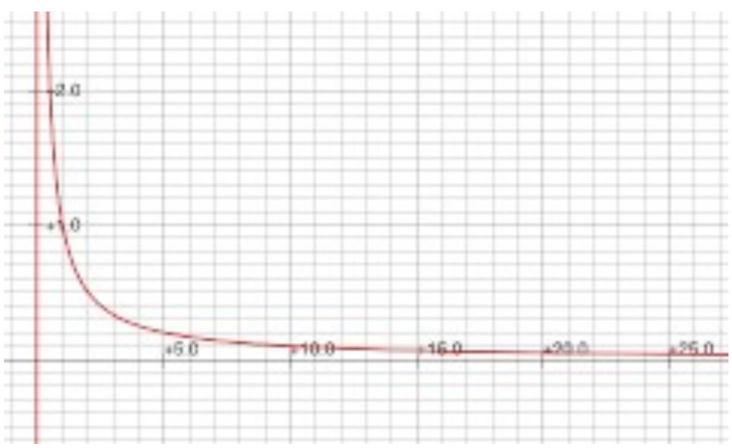
$$\lim_{x \to 0} f(x)$$

$$\lim_{x \to \infty} f(x)$$

If we have a limit like $\lim_{x \to 5} f(x)$ we can easily change it to $\lim_{h \to 0} f(5+h)$:

$$\lim_{x \to 5} f(x) = \lim_{h \to 0} f(5+h)$$

Graph of $y = \frac{1}{x}$



$$\lim_{x \to a} f(x) \qquad f(a)$$

When we say something like:

$$\lim_{h \to 0} \frac{\sin h}{h} = 1$$

We mean that,

as the values of h get closer and closer to 0

the value of $\frac{\sin h}{h}$ gets closer and closer to 1.

Here are some values:

$$\frac{\sin(.01)}{.01} = 1 - .0000166$$

$$\frac{\sin(.001)}{.001} = 1 - .0000000166$$

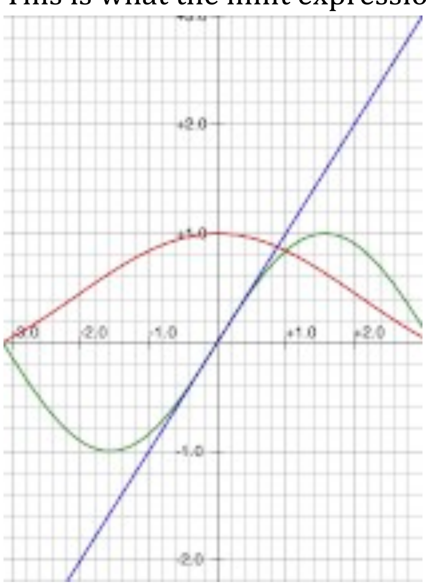
You use a limit here because if you plug in $h = 0$, you will have the undefined expression $\frac{0}{0}$.

We should say that,

as the value of h gets closer and closer to 0 but never is 0

the value of $\frac{\sin h}{h}$ gets closer and closer to 1.

This is what the limit expression means – it doesn't tell you why it is true.



Red is $y = \frac{\sin x}{x}$

Blue is $y = x$

Green is $y = sin\ x$

When you say something like:

$$\lim_{x \to \infty} \frac{x+2}{x} = 1$$

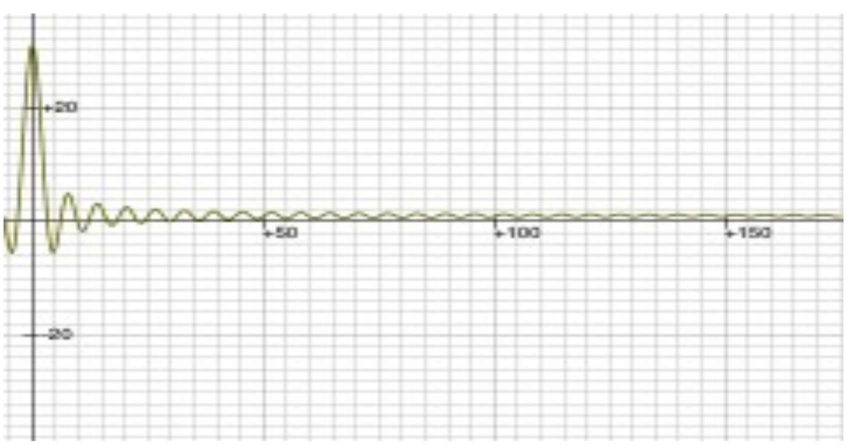
we mean that, as the value of x grows larger and larger with no bound, the value of $\frac{x+2}{x}$ gets closer and closer to 1.

We use a limit here because, if you try to plug in $x = \infty$ you will hit a roadblock. You can't do normal arithmetic with ∞.

This is what the limit expression means – it doesn't tell you why it is true.

Graph of $y = 1 + \left(\frac{30}{x}\right) * \sin(x)$

$$\lim_{x \to \infty} 1 + \left(\frac{30}{x}\right) * \sin(x) = 1$$



One more bit of notation.

Back to $y = 1/x$. As $x \to 0$, y gets bigger and bigger without any bound. We write this as

$$\lim_{x \to 0} \left(\frac{1}{x}\right) = \infty$$

Summary:

1. "Limit" is often written "Lim"
2. The basic limits are $\lim_{x \to 0} f(x)$ and $\lim_{x \to \infty} f(x)$
3. $\lim_{x \to a} f(x) = \lim_{h \to 0} f(a + h)$
4. We can have $\lim_{x \to a} f(x) = \infty$.
5. Limits are used when the values of $f(x)$ as x approaches a are not close to $f(a)$ or $f(a)$ doesn't exist.
6. Limits as $t \to \infty$ are used in mathematical models to calculate long-term expectations.
7. There is a rigorous formal definition of limit.

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